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Predictive ability of cognitive flexibility and planning in early mathematical competencies

Capacidad predictiva de la flexibilidad cognitiva y la planificación en las competencias matemáticas tempranas

Capacidade preditiva da flexibilidade cognitiva e do planejamento nas competências matemáticas precoces

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The dataset supporting the results of this study is not available.

Abstract: It has been shown that early mathematical competencies play an important role in the learning of this discipline and that both cognitive flexibility and planning favor this process. However, most research has related executive functions to general mathematical outcomes, without taking into account that this discipline includes several components that vary in their cognitive complexity. Therefore, the aim of this research was to evaluate the predictive capacity of cognitive flexibility and planning in the logical-relational and numerical mathematical competencies of 106 children in early childhood education in Chilean schools, who were evaluated with two executive tasks and a test of early mathematical competencies. Correlations and multiple linear regression models were used for data analysis. The results showed that cognitive flexibility was a significant predictor of both logical-relational and numerical mathematical competencies, while planning was only a significant predictor of numerical competencies. These results confirm the importance of cognitive flexibility and planning in the development of early mathematical competencies, which could lead to specific interventions on these executive functions and thus favor the learning of mathematics in early education.
Keywords: executive functions; cognitive flexibility; early math skills; early childhood education

Resumen: Se ha demostrado que las competencias matemáticas tempranas ejercen un rol importante en el aprendizaje de esta disciplina y que tanto la flexibilidad cognitiva como la planificación favorecen este proceso. Sin embargo, la mayoría de las investigaciones han relacionado las funciones ejecutivas con los resultados matemáticos generales, sin tomar en cuenta que esta disciplina incluye diversos componentes que varían en su complejidad cognitiva. Por lo tanto, el objetivo de esta investigación fue evaluar la capacidad predictiva de la flexibilidad cognitiva y la planificación en las competencias matemáticas lógico-relacionales y numéricas de 106 niños de educación inicial de escuelas chilenas, quienes fueron evaluados con dos tareas ejecutivas y un test de competencias matemáticas tempranas. Para el análisis de datos se realizaron correlaciones y modelos de regresión lineal múltiple. Los resultados mostraron que la flexibilidad cognitiva fue un predictor significativo de las competencias matemáticas tanto lógico-relacionales como numéricas, mientras que la planificación lo fue solo de las numéricas. Estos resultados confirman la importancia de la flexibilidad cognitiva y la planificación en el desarrollo de las competencias matemáticas tempranas, lo que podría propiciar intervenciones específicas sobre estas funciones ejecutivas y así favorecer el aprendizaje de las matemáticas en la educación inicial.
Palabras clave: funciones ejecutivas; flexibilidad cognitiva; competencias matemáticas tempranas; educación inicial

Resumo: Foi demonstrado que as competências matemáticas precoces desempenham um papel importante na aprendizagem dessa disciplina e que tanto a flexibilidade cognitiva como o planejamento favorecem esse processo. No entanto, a maioria das investigações relacionou as funções executivas com os resultados matemáticos gerais, sem ter em conta que esta disciplina inclui diversos componentes que variam na sua complexidade cognitiva. Por conseguinte, o objetivo desta investigação foi avaliar a capacidade preditiva da flexibilidade cognitiva e do planejamento nas competências matemáticas lógico-relacionais e numéricas de 106 crianças de escolas chilenas de educação infantil, que foram avaliadas com duas tarefas executivas e um teste de competências matemáticas precoces. Foram utilizados correlações e modelos de regressão linear múltipla para a análise de dados. Os resultados mostraram que a flexibilidade cognitiva foi um preditor significativo tanto das competências matemáticas lógico-relacionais como numéricas, enquanto o planejamento foi preditor significativo apenas das competências matemáticas numéricas. Estes resultados confirmam a importância da flexibilidade cognitiva e do planejamento no desenvolvimento de competências matemáticas precoces, o que poderia conduzir a intervenções específicas sobre essas funções executivas, favorecendo assim a aprendizagem da matemática na educação infantil.
Palavras-chave: funções executivas; flexibilidade cognitiva; competências matemáticas precoces; educação infantil

It is well known that Early Mathematical Competencies (EMC) perform an important role for education as well as for human development. This, due to their usefulness in the performance of diverse daily life activities and in the adaptation to environmental challenges (Fisk & Lombardi, 2021; Limas et al., 2020).

Conceptually, EMCs include the abilities to use, assess and understand mathematics in different necessary contexts (Cerda et al., 2011; Cerda et al., 2012; Raghubar & Barnes, 2017). At the same time, these skills incorporate an interactionist view regarding mathematics learning during childhood, in the sense that they assume that both logical and counting skills are responsible for the development of number sense in children (Cerda et al., 2012). From this perspective, EMCs can be classified into two big groups: (1) those of logical-relational type, which include the comparison, classification, correspondence and seriation abilities, and (2) those of numeric type, that include the abilities of verbal counting, structured counting, resultant counting and general knowledge of numbers (Cerda & Pérez, 2015; Van de Rijt et al., 1999).

Due to the previously mentioned and considering that the EMCs are the fundamental basis for the acquisition and development of more advanced mathematical skills at later stages of development (Purpura et al., 2017), is it essential to stimulate them precociously, specifically during the early childhood education period, since it is a proper stage to strengthen and enhance their development (Cerda et al., 2011).

Despite their importance, the EMCs assessment is not frequent in early childhood education, mainly because, in Chile, there are few standardized tools to perform it (Cerda et al., 2012). Most assessments are conducted for elementary and secondary school students (Cerda et al., 2012), stages where learning difficulties are already evident and hard to solve (Agencia de Calidad de Educación, 2020; Chu et al., 2016).

In addition, several studies have shown that approximately 20% of the child population presents some type of mathematical learning difficulty (Moll et al., 2014; Wang et al., 2018). This fact is corroborated by the results obtained in Chile, in the TIMSS (Trends in International Mathematics and Science Study) test, which is the international assessment that measures the academic performance in Mathematics and Science of 8th and 12th grade students (Fishbein et al., 2021). According to 2019 results, Chilean students present a below-average performance, where from 18% to 30% does not have the basic knowledge of the assessed areas (Agencia de Calidad de Educación, 2020; Arias, 2020).

Against this background, the importance of EMCs assessment in early childhood education is confirmed, not only because of its significant contribution to positive outcomes in the further academic development of children in school (ten Braak et al., 2022), but also because it is considered that they can be a mean to improve a board set of skills such as mathematics, language and the executive functions (Mattera et al., 2017)

In this regard, many authors (Purpura, 2017; ten Braak et al., 2022; Viterbori et al., 2015; Yang et al., 2019) have explored the role of executive functions (EFs) in the EMCs development. The EFs are considered essential for mathematics learning (Arán Filippetti & Richaud, 2017; Escobar et al., 2018; ten Braak et al., 2022), mainly for their contribution in processing information to achieve an objective and solve problems (Chan & Scalise, 2022).

EFs are a family of higher order processes that make it possible to pay and maintain attention, solve problems, adapt to new situations, have self-control and discipline, see things from different perspectives and adjust to change (Diamond, 2020; Tirapu-Uztárroz et al., 2018). Hence its importance in learning and school success (Bernal-Ruiz et al., 2020).

There are two theoretical perspectives regarding EFs dimensionality. The first one refers to them as a unitary construct (Hughes et al., 2010) and the second one states that EFs are multidimensional, this means that, although they are independent processes, they are interrelated with each other (Tirapu-Uztárroz et al., 2018). The latter perspective distinguishes at least 3 main executive domains: inhibition, working memory and cognitive flexibility (Miyake et al., 2000; Salehinejad et al., 2021). Inhibition refers to the ability to suppress an overbearing response in favor of a more adaptive one (Coulanges et al., 2021). Working memory refers to the capability of maintaining, manipulating and transforming information while performing a task (Allen et al., 2021). Meanwhile, cognitive flexibility refers to the ability to simultaneously consider different options and to change attention in a flexible way between them (Chan & Scalise, 2022; Rosas et al., 2017).

From these 3 executive domains derive planning and problem solving (Diamond, 2020), which are related to thinking and behavior organization, succeeding in anticipating consequences and creating and selecting options and mind maps to direct action towards a goal or objective (Arroyo et al., 2014; Deng et al., 2020; Diamond, 2020; Díaz et al., 2012).

This study is specifically focused on EFs of cognitive flexibility and planning and its contribution to mathematical achievement in early childhood education.

Cognitive flexibility (CF) is related to the ability of modifying a learned mental scheme according to environmental demands (Chan & Scalise, 2022; Legare et al., 2018; Rosas et al., 2017; Van der Ven et al., 2011). In this context, for Diamond (2020), CF has two subcomponents, one of them entails the capability of seeing something from different perspectives, and the other one, the ability to adapt quickly and flexibly to changes in order to find different ways to reach a desired goal. In other words, this EF allows the change of attentional focus between diverse solving strategies, not only considering divergent variables for problem solving, but also cancelling automatic behaviors (Cantin et al., 2016; Chan & Scalise, 2022; Nunes de Santana et al., 2022).

From the anatomical point of view, CF is known to emerge from the prefrontal cortex (PFC) and is connected to the basal ganglia (Cameron et al., 2010; Chakravarthy et al., 2010; Pauli et al., 2016; Stocco et al., 2010; Zink et al., 2021). Indeed, Zink et al. (2021) recognize that CF is connected to specific brain areas anatomically confined to PFC and can be related to a set of similar or overlapping brain structures. In this sense, CF would activate big brain portions, converging in PFC the anterior cingulate cortex and the posterior parietal cortex (Chakravarthy et al., 2010; Niendam et al., 2012; Stocco et al., 2010; Zink et al., 2021).

From this background, many authors consider that CF favors mathematical problem solving, and even, stands as an achievements and mathematical performance predictor during the stage of early childhood and school education (Cantin et al., 2016; Magalhães et al., 2020; Nunes de Santana et al., 2022; Palacios & Bohlmann, 2020). Achievements that are possible due to the ability of perspective changing, adapting to a changing environment, and having a divergent thinking (Ropovik, 2014; Titz & Karbach, 2014).

In the same vein, to Cheung and Chan (2022), CF would play a crucial role for both mental calculation execution and complex mathematical problem solving. This, because the nature of these operations requires a constant transition between different cognitive processes that allow processes to be carried out, such as the problem mental representation, the relevant information integration, a resolution plan design and the execution of this plan, which are essential for mathematical problem solving (Cheung & Chan, 2022; Mayer & Hegarty, 1996).

On the other hand, as stated previously, the Planning allows not only the formulation of action plans, but also their execution and their effectiveness assessment (Cortés et al., 2019). This executive function has been associated with the frontal lobe functioning, specifically with the lateral prefrontal cortex and the dorsal caudate nuclei, which play an important role in selecting cognitive abilities appropriate for the formation of goals and objectives and in the development of necessary plans to achieve them. (Cai et al., 2016; Levy & Dubois, 2006; Luria, 1966; Turnbull, 2002). Thence its link with mathematical performance (Agudelo et al., 2016).

Indeed, although the relation between Planning and mathematical competencies has been addressed, it has not been as studied as the other EFs, consequently, its contribution has not been fully established yet (Arroyo et al., 2014). However, results from some studies suggest that Planning plays an important role in the development of mathematical skills. In fact, Cai et al. (2016) demonstrated this role by pointing out that Planning exerts influence during the mathematics learning process and that this process occurs independently of the others EFs such as working memory, consequently, for these authors, paying attention to this executive domain since early childhood education increases the chances of detecting children with mathematical difficulties.

In short, there are multiple studies that have addressed the relation between EMCs and EFs, however, most of these studies have focused only on the three main executive domains (i.e., working memory, cognitive flexibility and inhibition) (Van der Ven, 2011; Viterbori et al., 2015), relating them to general mathematical results, without taking into account that this discipline includes many components, which vary depending on their cognitive complexity and, thereby, is important to know EFs that underlie each of them specifically. In this respect, Viterbori et al. (2015) point out that a comprehensive model that includes aspects of executive functioning and different components of mathematical skills during the early stages of mathematical learning is necessary.

In the basis of the foregoing, it is worth asking; Are the EFs of Planning and CF predictors of early mathematical competencies in early childhood education children? Based on this, this study intends to provide new results to existing research, in order to complement and deepen in the study of predictive capacity of CF and Planning domains in EMCs, seeking to provide useful information to the field of knowledge on the topic.

Having said that, considering the theoretical framework that currently exists about this topic, an observation of a predictive capability of the two executive domains (CF and Planning) is expected that is statistically significant on the EMCs development. In other words, predictive capability of both CF and Planning is expected to be statistically significant in the development of logical-relational mathematical competencies and also in the numeric ones of early childhood education children.

The main objective of this study was to determine the predictive capability of CF and Planning in early mathematical competencies of logical-relational (Logical-Relational MC) type (comparison, classification, correspondence, seriation) and numeric (Numeric MC) type (verbal counting, structured counting, resultant counting, and general knowledge of numbers) in early childhood education children.

Method

Research design

An ex post facto non-experimental design was applied, which aims to examine the effects of a natural event over a subsequent result, with the objective to establish a causal or correlational association between them (Kerlinger & Lee, 2000; Portell & Vives, 2019). In this case, the predictive capability of CF and Planning was established in the development of the EMCs dimensions of early childhood education children.

Participants

The sampling type was non-probabilistic for convenience and was composed by 106 early childhood education students (males n = 47, 44.3%; females n = 59, 55.7%), from public (n = 20, 18.9%), subsidized (n = 72, 67.9%) and private (n = 14, 13.2%) educational establishments from the Valparaíso region of Chile. Fifty students were from prekindergarten (47.2%) (males n = 22, *average age* = 5.01; females n = 28; *average age* = 4.89) and 56 were from kindergarten (52.8%) (males n = 25, *average age* = 6.05; females n = 31; *average age* = 5.96).

The inclusion criteria were: a) attending prekindergarten or kindergarten, b) having their families authorization to participate in the research through the signing of an informed consent. The exclusion criteria were: a) present any neurodevelopmental disorder, b) be under pharmacological treatment that may affect the executive functions performance, and c) not wanting to participate in the study or not having the informed consent duly signed by their families.

Instruments

Mathematical competencies of participants were assessed by the *Utrecht Early Mathematical Assessment Test* (Cerda et al., 2012), which assesses eight competence areas of EMCs, four for logical-relational dimension (i.e., comparison, correspondence, classification, and seriation) and four for numeric dimension (i.e., verbal counting, structured counting, resultant counting and general knowledge of numbers). This test is applied to children between 4 and 7 years old and lasts from 20 to 30 minutes approximately. It consists of 40 items assembled in the 8 competence areas that include 5 items each. Consequently, it has a maximum score of 40 points, of which 20 points belong to logical-relational type EMCs and 20 to numeric type ones. Its reported Cronbach's alpha is .91 (Cerda et al., 2012).

For CF assessment, the *Dimensional Change Card Sort* (DCCS) test (Zelazo, 2006) was used, in which participants classify bivalent cards (e.g. red trucks and blue stars) based on an attribute (e.g. by color) and then, they are asked to change and classify the same cards based on a new attribute (e.g. by shape). This test is applied to children from 3 to 5 years old and lasts approximately 5 minutes. Its Cronbach's alpha is .94 (Zelazo, 2006).

The Planning was assessed using the *Porteus Maze Test* (Porteus, 1965), that measures a person's ability to develop and carry out an action plan. This instrument can be administered in children from 3 years old, it is administered individually and lasts approximately 10 minutes. It has 12 mazes of increasing difficulty, in which the child must locate and draw with a pencil the shortest route from the beginning to the end of the maze without entering a dead-end street. This instrument has an adequate internal consistency with a Cronbach's alpha of .81 (Krikorian & Bartok, 1998).

Procedure

Firstly, a meeting was held with the management team of each educational establishment that accepted to be part of the study, where they were explained the study objective and, at the same time, they were asked to hand the informed consent to the families in order that they could sign the authorization for the children's participation.

Then, children whose families authorized their participation were individually assessed in a session of approximately 45 minutes during their school day. Assessments were carried out between September and October 2022.

Data analysis plan

Firstly, descriptive analyses to synthesize sample demographic information were carried out. Subsequently, correlation analyses to establish association between participants' CF, Planning and EMCs were made. Finally, multiple hierarchical linear regression models for the assessment of EFs predictive capability over children EMCs dimensions were made. Analyses were made with the Jamovi statistical software, 2.2.5 version (The jamovi project, 2021).

Ethical considerations of human being research

All procedures were implemented following the Singapore Statement on Research Integrity (Comisión Nacional de Investigación Científica y Tecnológica, 2010) guidelines. This is why an informed consent protocol was defined, that was signed by the sample children 's guardians, as well as an assent which was shown to participants. Additionally, the study met with the approval of the University 's Ethics Committee.

Results

Descriptive analyses

Table 1 shows, for each grade, the average values and standard deviation of all study variables.

Table 1
Descriptives of variables under study

	Grade	*N*	*M*	*SD*
Cognitive Flexibility	Prekindergarten	50	8.420	5.675
	Kindergarten	56	10.357	4.886
Planning	Prekindergarten	50	136.240	31.394
	Kindergarten	56	145.714	24.839
Logical-Relational MCs	Prekindergarten	50	11.720	4.010
	Kindergarten	56	14.268	3.194
Numeric MCs	Prekindergarten	50	7.760	4.396
	Kindergarten	56	11.750	4.818

Correlation analysis between cognitive flexibility, planning and early mathematical competencies

Firstly, to assess the predictive capability of CF and Planning over Logical-Relational MCs and Numeric MCs of the sample's children, the univariate assumption of normality was analyzed for the Pearson correlation coefficient, which was fulfilled. Subsequently, correlation matrix was generated between the children 's Logical-Relational MCs, Numeric MCs, CF, and Planning.

Significant correlations between all variables (Table 2) were found. In the case of Logical-Relational MCs, the correlation with Planning is moderate (.406), and with CF it is rather discrete (.278). Additionally, significant correlations were found regarding Numeric MCs, with CF (.348) and Planning (.263).

Table 2
Correlation matrix of mathematical competencies with cognitive flexibility and planning

		Executive Functions	
		Cognitive Flexibility	Planning
Logical-Relational MCs	Pearson's r	.278**	.406***
	p value	.004	< .001
Numeric MCs	Pearson's r	.348***	.263**
	p value	<.001	0.006

** $p < .01$ *** $p < .001$

Multiple linear regression models: CF and Planning as predictors of children's mathematical competencies

In order to know which EF (i.e., CF and Planning) predicts EMCs (i.e., Logical-Relational MC and Numeric MC) of the participants, a multiple hierarchical linear regression analysis was applied, where the explanatory variables were introduced as predictors according to the degree of correlation with the dependent variables of Logical-Relational MCs and Numeric MCs. Given this, the executive domains that showed higher correlation with the EMCs were added in each regression model.

In Table 3 the tested models are presented, and it can be observed that Planning is the best predictor of Logical-Relational MCs, since as a single predictor it explains 16.5% of Logical-Relational

MCs variability. By adding the CF to the model, as a second predictor, it explains 21.3% and when the Course is added as a factor, the model explains 27.1% of the variability of children's scores in the Logical-Relational MCs. Therefore, a three predictor model composed by Planning, CF and Course allows to significantly predict Logical-Relational MCs, $R^2 = 0.271$, F (3, 102) = 12.6, $p < .001$.

Meanwhile, CF predicted the Numeric MC in a better way, since as a single predictor, it explains 11.9% of Numeric MCs variability. By adding Planning as a second predictor, it explains 16.6%, and adding the Course as a factor, the model explains 26.4% of variability of children's scores in the Numeric MCs, however, in this last case, Planning is no longer a significant predictor ($p = .051$) in such model (see Table 4). Hence, only a two predictors model was left, composed by CF and Course, which allows to significantly predict the Numeric MCs, $R^2 = 0.237$, F (2, 103) = 16.0, $p < .001$, explaining 23.7% of variability of children's scores in the Numeric MCs.

It is important to note that as in both models, statistically significant differences were observed for the course factor (Table 4) - The same didn't happen for gender nor type of educational establishment-, the regression equations introduce the dichotomized course variable (i.e., fictitious variable), which takes the value 0 if the participant attends prekindergarten and the value 1 if she or he attends kindergarten.

Thus, the regression equation for the Logical-Relational MC variable would be: [$y = \alpha + (\beta_1$*Planning) + (β_2*CF) + (β_3* dichotomized course) + ε], of which values are: [$y_i = 4.441$ + (0.045* Planning$_i$ score) + (0.131 CF$_i$ score) + (1.865* course$_i$) + ε]. The subscript "*i*" indicates the person of interest. On the other hand, the regression equation of Numeric MCs would be: [$y = \alpha + (\beta_1$*CF) + (β_2* course) + ε] of which values are: [$y_i = 1.564$ + (0.246*CF$_i$ score) + (3.226* course$_i$) + ε]. In both equations, γ corresponds to the criterion value, α to the intercept estimated value, β_1, β_2 and β_3 to the estimate of the unstandardized regression coefficients of the variables (i.e., Planning, CF, and Course) and ε to the standard error of prediction.

Table 3
Summary of the multiple hierarchical linear regression models tested to predict.

Model	R	R^2	Adjusted R^2	Exchange Rate Statistics		gl_1	gl_2	Rate of change of P at F
				R^2 Exchange	F Exchange			
Logical-Relational Mathematical Competencies (Logical-Relational MC)								
1	.406[a]	0.165	0.157	–	20.5	1	104	–
2	.462[b]	0.213	0.198	0.049	6.3	1	103	.013 *
3	.520[c]	0.271	0.249	0.057	7.9	1	102	.006 **
Numeric Mathematical Competencies (Numeric MC)								
1	.345[d]	0.119	0.111	–	14.0	1	104	–
2	.407[e]	0.166	0.149	0.046	5.7	1	103	.019 *
3	.514[f]	0.264	0.242	0.098	13.6	1	102	<.001 ***

Notes. [a] Predictor: Planning. [b] Predictors: Planning and Cognitive Flexibility. [c] Predictors: Planning, Cognitive Flexibility and Course. [d] Predictors: Cognitive Flexibility. [e] Predictors: Cognitive Flexibility and Planning. [f] Predictors: Cognitive Flexibility, Planning and Course.
*$p < .05$ ***$p < .001$

Table 4
Regression models of cognitive flexibility and planning predicting children's mathematical competencies

Dependent Variable	Predictor	Model Coefficients: Unstandardized regression coefficients	Model Coefficients: Standardized regression coefficients (β)	Model Coefficients: t	Model Adjustment: *p* Value	Model Adjustment: R^2	Model Adjustment: ΔR^2	Collinearity: VIF[a]
Logical-Relational Mathematical Competencies	Intercept	4.441	--	2.66	.009 **	--	--	--
	PLA	0.045	0.338	3.91	<.001 ***	0.165	--	1.04
	CF	0.131	0.184	2.12	.036*	0.213	0.049	1.04
	Course	1.865	0.490	2.82	.006 **	0.271	0.057	1.05
Numeric Mathematical Competencies	Intercept	1.564	--	0.78	.481	--	--	--
	CF	0.246	0.262	3.01	.003 **	0.119	--	1.04
	PLA	0.030	0.171	1.97	.051 *n.s*	0.166	0.046	1.04
	Course	3.226	0.643	3.69	<.001 ***	0.264	0.098	1.05

Notes. PLA: Planning; CF: Cognitive Flexibilit; *n.s*: not significant. [a]. *Variance inflation factor*.
**p* <.05 ***p* < .01 ****p* <.001

Discussion

The objective of this study was to assess the predictive capability of the executive functions of CF and Planning in early mathematical competencies in early childhood education children. The findings show that both functions have a predictive capability in early mathematical competencies. The CF in both logical-relational and numeric types, and Planning only in logical relational ones. In this respect, the expected was partially fulfilled, since the hypothesis was that the predictive capability from both executive functions would be significant respecting the development of both types of mathematical competencies in the sample's children.

Despite the above, the findings are in line with the researches results that propose that the CF is related to mathematical performance in early childhood education children (Yang et al., 2019) and other studies such as the ones from Cheung and Chan (2022) and Palacios and Bolhmann (2020), who concluded that CF is a predictor for the development of mathematical competencies also in later stages of schooling. This demonstrates that CF is essential not only for the change of focus in the mental representation of a problem and the incorporation of relevant data about it, but also for the creation of a solution plan and its execution, abilities that are fundamental for solving mathematical problems (Cheung & Chan, 2022; Mayer & Hegarty, 1996). Likewise, Nunes de Santana et al. (2022) concluded that the CF influence is present in both mathematical competencies; those linked to piagetian abilities and those of numeric domain, which is confirmed by the findings of this study.

Regarding the finding of the Planning influence in logical-relational mathematical competencies (Logical-Relational MC), it is believed that this could respond to the own evolutionary process of these mathematical skills, since it is known that they develop at an early age, even before the schooling process, coinciding with the preoperative stage (Cerda et al., 2011; Piaget, 1965), in a way that preschoolers have had more experience using planning in tasks involving comparison, correspondence, classification and seriation, than in numeric type tasks (Bernal-Ruiz et al., 2022).

Similarly, this finding is in line with other studies that have placed Planning as a good predictor of mathematical performance in child population, concluding that children efficiency to solve mathematical tasks is not only related to the mathematical ability they may have, but also it depends on their ability to self-regulate and plan themselves (Agudelo et al., 2016; Arroyo et al., 2014; Tzuriel et al., 2022), which confirms that processes such as formulation, execution and assessment of an action plan are necessary for mathematics learning (Bernal-Ruiz et al., 2022).

In this same perspective, and regarding to the non-significant influence of Planning on Numeric MC, this finding coincides with the recent research of Tzuriel et al. (2022), where in children between 3 and 6 years old, Planning was not a significant predictor of numeracy-related mathematical performance either. This can be attributed to the later numeric competencies development compared to logical-relational ones in childhood, since the first ones are developed in later stages than the preoperational one (Cerda et al., 2011), and in this sense, they would require the development of Logical-Relational MCs to be acquired (Cerda et al., 2011; Piaget, 1965).

In the same vein, it is generally known that schooling favors the development of Numeric MCs, as a result of the traditional numbering system teaching that occurs in that context (Cerda et al., 2011). Therefore, as the sample is composed of children from 4 to 6 years old, it is believed that it is possible that they present an early development in this type of competence, since they are in the process of adapting to the educational system. This adds the Planning evolutionary development, which is progressively formed from 4 years old, reaching the full development at 15 years old (Injoque-Ricle et al. 2014; Muchiut, 2019; Rubiales et al., 2011) due to the frontal lobe maturation throughout the life cycle (Er-Rafiqi et al. 2022; Segundo-Marcos et al. 2022), structure that completes its development at the end of childhood.

Although this study provides interesting findings, especially regarding the predictive capability of a relatively neglected EF, such as Planning in the development of EMCs, it presents some limitations. Firstly, the sampling size was limited (106 children) and the sample type was intended, which restrains the possibility of making generalizations. Consequently, it is suggested for future studies to expand the sampling size. Secondly, working memory was not considered as a possible predictor variable, even though there is significant evidence that it is one of the EFs most associated with mathematical abilities and numeric comprehension (Allen et al., 2021; Bisagno et al., 2023; Peng et al., 2016). Additionally, it would be interesting to include in future studies other EFs, such as inhibitory control, which is also considered an important executive domain in the childhood mathematical performance (Cueli et al., 2020).

Despite these limitations, this study constitutes a contribution for research regarding the relation of EFs of CF, Planning and mathematical performance in early ages. Additionally, it stands as a novel contribution to research, specifically of Planning, which has been poorly studied and that in light of this study's findings is constituted as a predictor variable of early mathematical competencies, specifically those of logical-relational type.

Finally, based on the results of this study, the aim is to provide teachers with useful information that may serve as an input for the development of curricular plans focused on strengthening the mathematics area, specially by stimulating the EMCs development, as well as diverse researches have not only evidenced that these mathematical competencies constitute a strong and stable predictor of achievement of the academic performance in mathematics and other disciplinary areas (Cerda & Pérez, 2015), but also allow the continuous acquisition of more complex mathematical knowledge and abilities in later schooling stages (Purpura et al., 2017). In addition, it is expected that this study can guide both caregivers and teachers of early childhood education, in order that they develop stimulation strategies of EFs, CF and Planning in-classroom and out-of-classroom, since it has been demonstrated that children that start early childhood education with better executive abilities have an advantage in terms of mathematical performance that lingers over school years (Arroyo et al., 2014; Cervigni & Stelzer, 2011; Clements et al., 2016; Wongupparaj & Kadosh, 2022).

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